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Magnetar field dynamics shaped by chiral anomalies and magnetic helicity

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MAgneto-Thermal evolution of Isolated Neutron Stars

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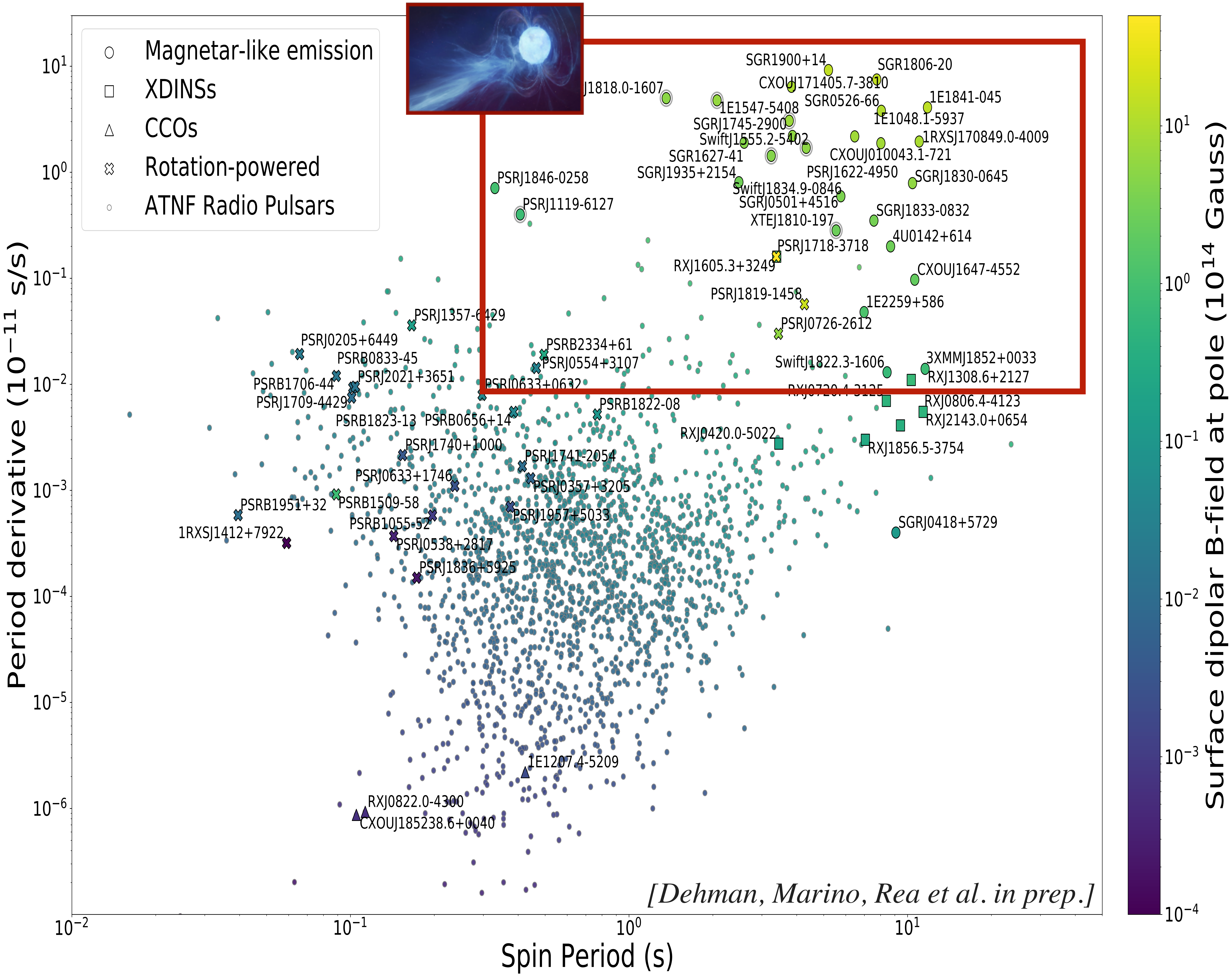
How does the large-scale dipolar field form in magnetars?



Magnetars

- Powered by magnetic field dissipation ($B_{avg} \approx 10^{15} \dots 10^{16}$ G)
- Young objects ($t \lesssim 10$ kyr).
- Slow rotator ($P \sim 1 \dots 12$ s).
- Bright X-rays sources:
- $L_x \sim 10^{34} \dots 10^{36}$ erg/s
- Characterised by outbursts and flares

$$B_{dip} = 6.4 \times 10^{19} \sqrt{P\dot{P}}$$

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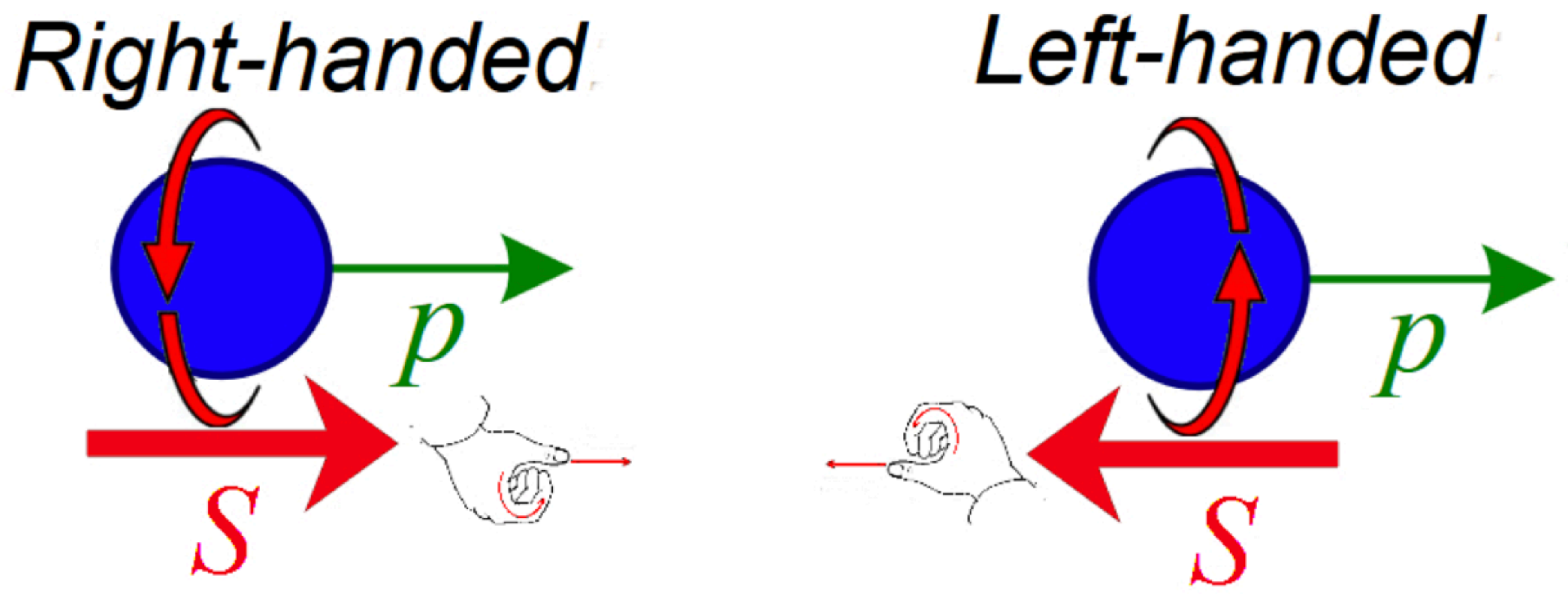
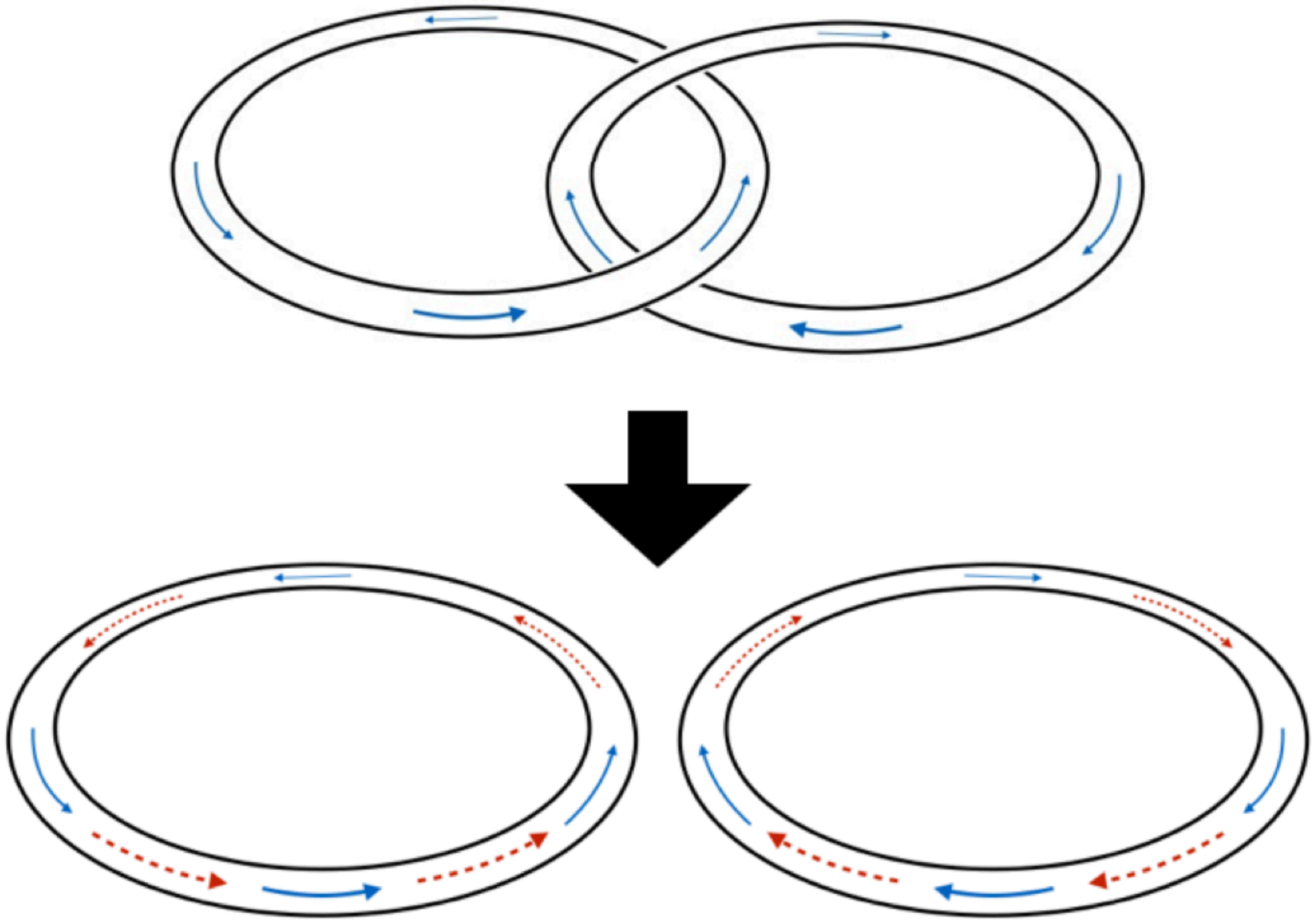
Chiral Anomaly



A particle is moving with momentum p represented by the green arrow.



[Boyarsky et al. 2012]

Image credit: Y. Hirono

— Particle Chirality —

The chirality of a particle is the projection of the spin along the direction of movement:

- right-handed if it is parallel to the movement;
- left-handed otherwise.

— Magnetic Helicity —

- Pure poloidal & toroidal components are unstable
- Both components are necessary for the stability (linked structures)
- Magnetic helicity quantify the topological stability

— Chiral Anomaly —

- Changes in magnetic helicity create or destroy chiral asymmetry (vice versa).
- Chiral electric current induced along field lines

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Evolution of the chiral number density n_5

Chiral number density evolution $n_5 \approx \mu_e^2 \mu_5 / \pi^2 (\hbar c)^3$

$$\frac{\partial n_5}{\partial t} = \boxed{\frac{2\alpha}{\pi\hbar} \boldsymbol{E} \cdot \boldsymbol{B}} + n_e \Gamma_w^{\text{eff}} - n_5 \Gamma_f$$

In the absence of an external source term, the magnetic field itself serves as a source of the chiral asymmetry.

Γ_w^{eff} effective weak interaction rate acts as a source term when the star is out of chemical equilibrium.

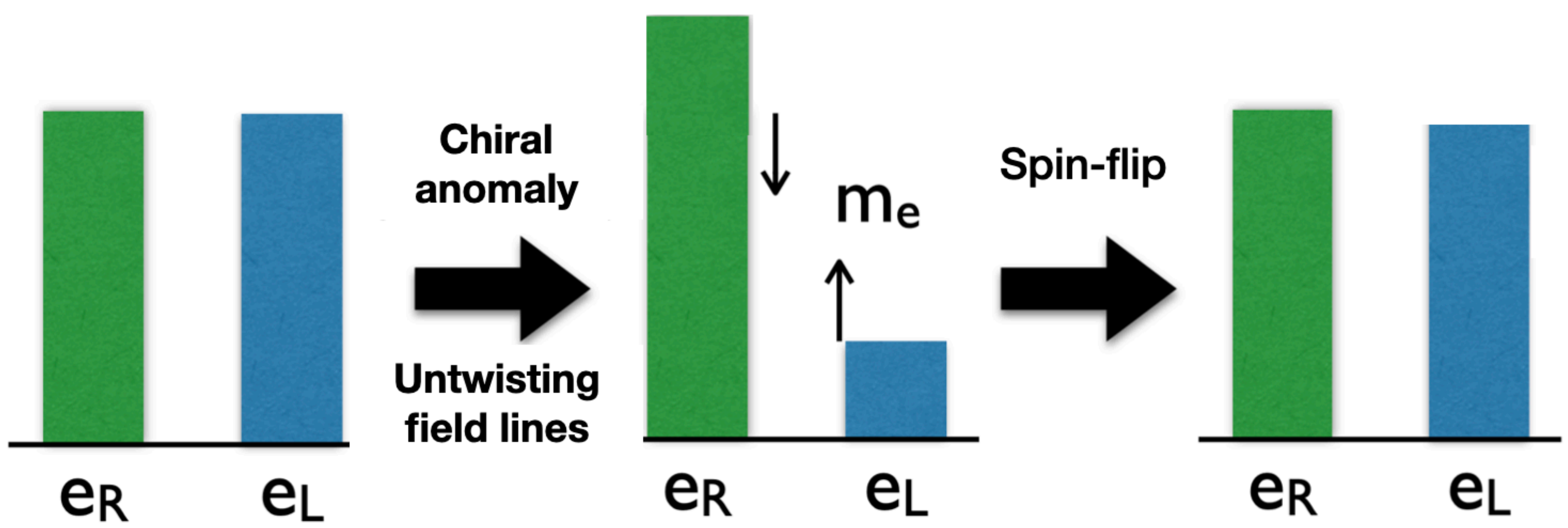
Γ_f spin-flip rate from finite electron mass (EM interactions), acting as a sink term.

$\boldsymbol{E} \cdot \boldsymbol{B}$ couples chiral density to the EM field: twisting or untwisting magnetic lines changes net chirality, acting as a source or sink depending on its sign.

The flip term depends on temperature through the crust's electrical conductivity inside the neutron star:

$$\Gamma_f = \left(\frac{m_e}{\mu_e}\right)^2 \nu_{\text{coll}} = \frac{4\alpha}{3\pi\sigma_e} \frac{m_e^2 c^4}{\hbar^2}$$

Despite the strong suppression of the chiral asymmetry, it remains relevant on neutron star timescales



Inspired by an image from N. Yamamoto

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[Dehman & Pons, 2505.06196]

Generalized helicity balance law

Chiral number density evolution $n_5 \approx \mu_e^2 \mu_5 / \pi^2 (\hbar c)^3$

$$\frac{\partial n_5}{\partial t} = \boxed{\frac{2\alpha}{\pi\hbar}\boldsymbol{E}\cdot\boldsymbol{B}} + n_e\Gamma_w^{\text{eff}} - n_5\Gamma_f$$

Time evolution of the magnetic helicity:

$$\frac{\partial(\boldsymbol{A}\cdot\boldsymbol{B})}{\partial t} = \boxed{-2c\boldsymbol{E}\cdot\boldsymbol{B}} - c\nabla\cdot(\boldsymbol{E}\times\boldsymbol{A})$$

Generalised Helicity Balance Law

(Combined and volume-integrated form of both equations)

$$\frac{d}{\partial t}\left(Q_5 + \frac{\alpha}{\pi\hbar c}\chi_m\right) + \Gamma_5 = 0$$

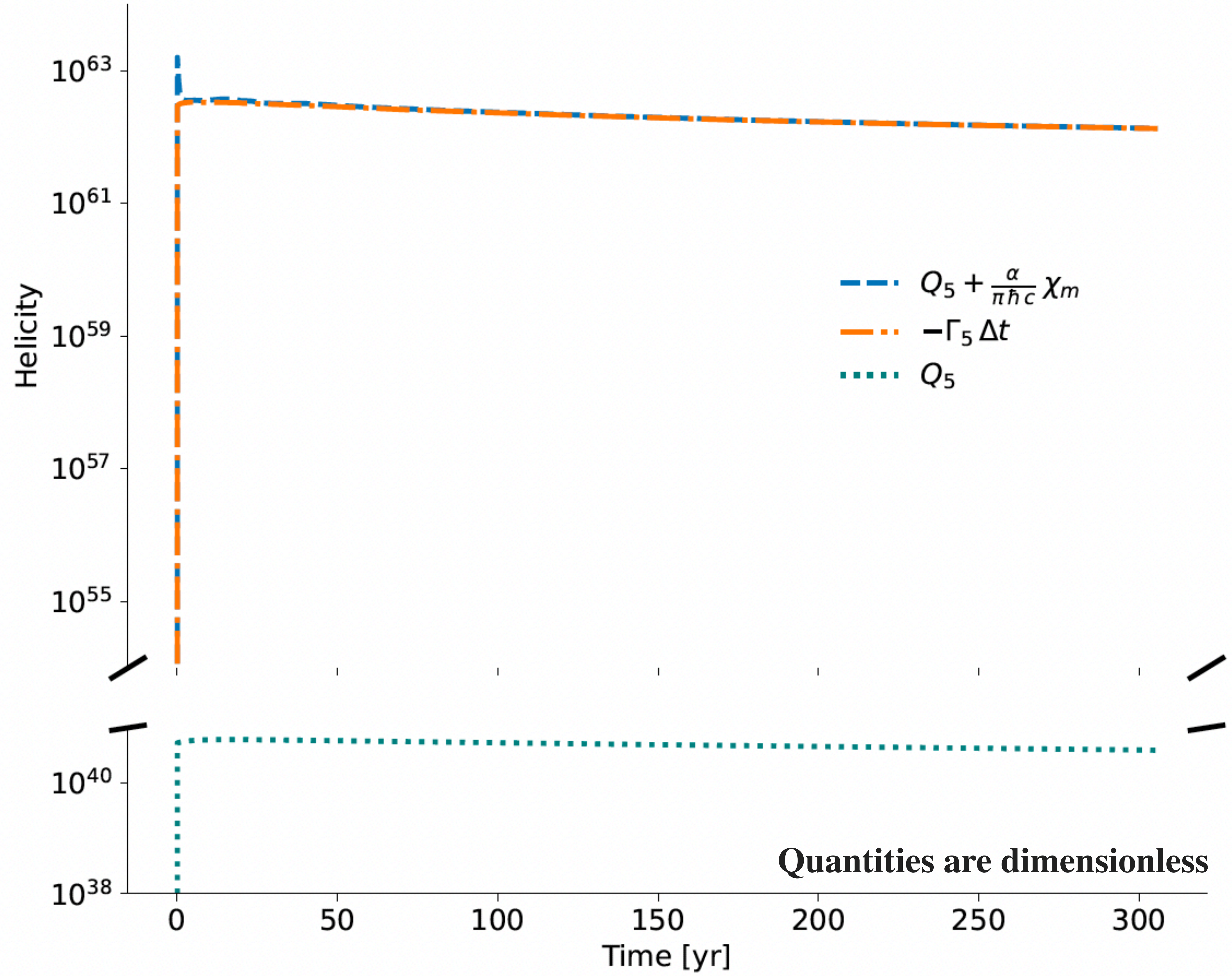
$$\chi_m = \int \boldsymbol{A}\cdot\boldsymbol{B}\, dV, \qquad Q_5 = \int n_5\, dV, \qquad \Gamma_5 = \int n_5\Gamma_f\, dV$$

Total helicity is no longer conserved.

Its time evolution is governed by the average spin-flip rate.

[Dehman & Pons, 2505.06196]

Magnetic field alone serves as a source of the chiral asymmetry.



$$\mu_5 \approx 10^{-12}\ldots10^{-11}\ \text{MeV} \ll \mu_e = 10\ldots100\ \text{MeV}$$

Within the Standard model, you should account for the chiral asymmetry in the presence of magnetic helicity.

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Chiral current J_5

In the presence of a magnetic field B and a non-vanishing chiral chemical potential:

$$\mu_5 = \mu_R - \mu_L \neq 0$$

$$\left(k_5 = \frac{4\alpha\mu_5}{\hbar c} \right)$$

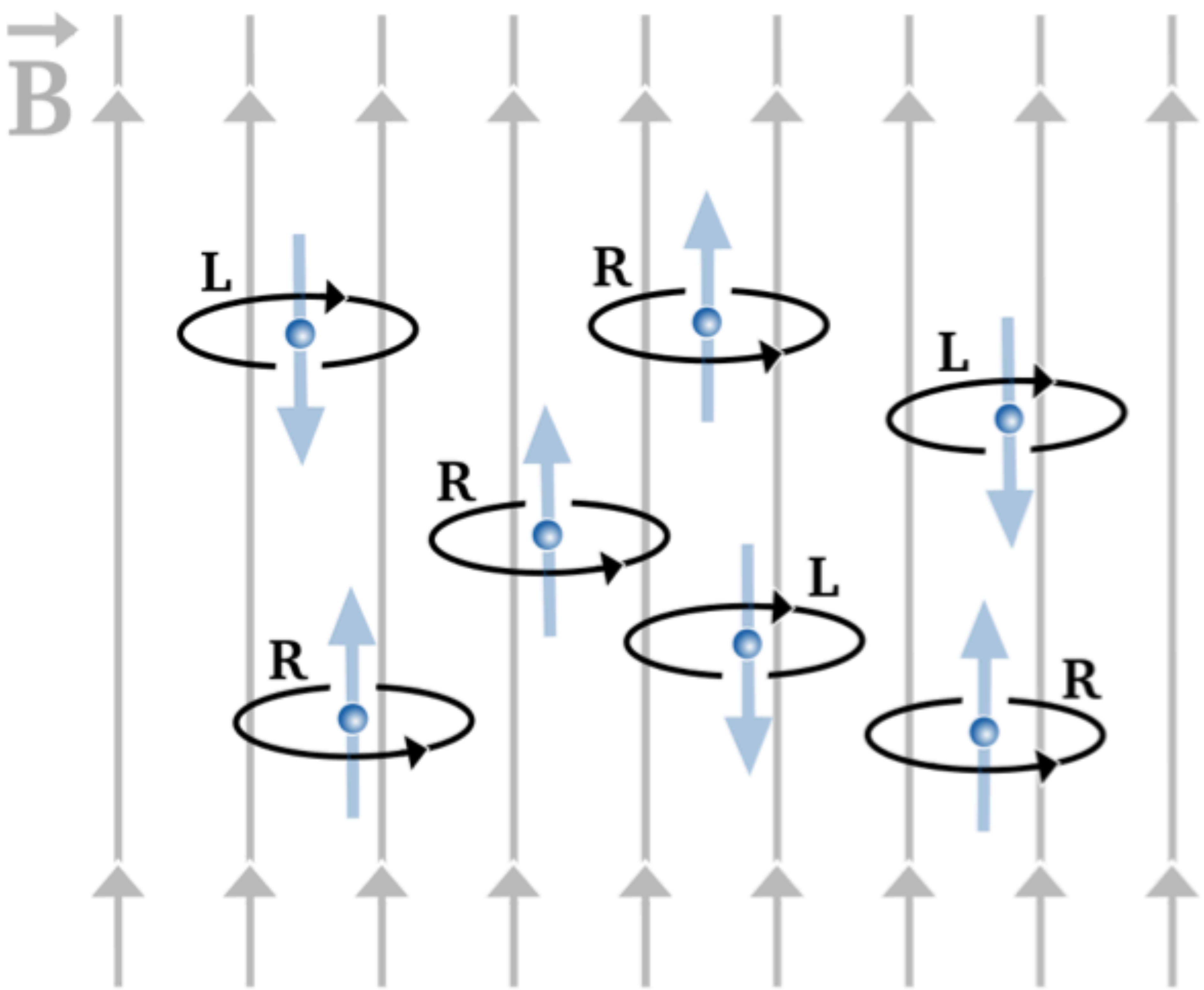

Image credit: J. Schober

Right-handed:

$$\boldsymbol{J}_R = \frac{\alpha\mu_R}{\pi\hbar}\boldsymbol{B}$$

Left-handed:

$$\boldsymbol{J}_L = -\frac{\alpha\mu_L}{\pi\hbar}\boldsymbol{B}$$

$$\boldsymbol{J}_5 = \frac{\alpha\mu_5}{\pi\hbar}\boldsymbol{B}$$

Chiral electric current density in the direction of the magnetic field

$$\mu_5 \approx 10^{-12}\ldots10^{-11}\ \text{MeV} \ll \mu_e = 10\ldots100\ \text{MeV}$$

$$\boldsymbol{J} = \frac{c}{4\pi}(\nabla \times \boldsymbol{B}) = \sigma_e \boldsymbol{E} + \boldsymbol{J}_5$$

→ Modified induction equation.

[A. Vilenkin 1980]

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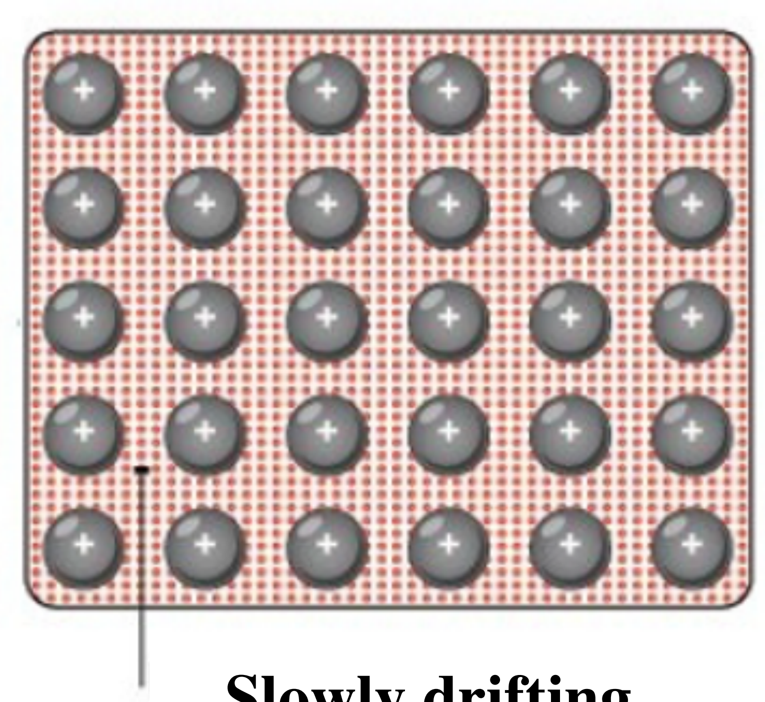
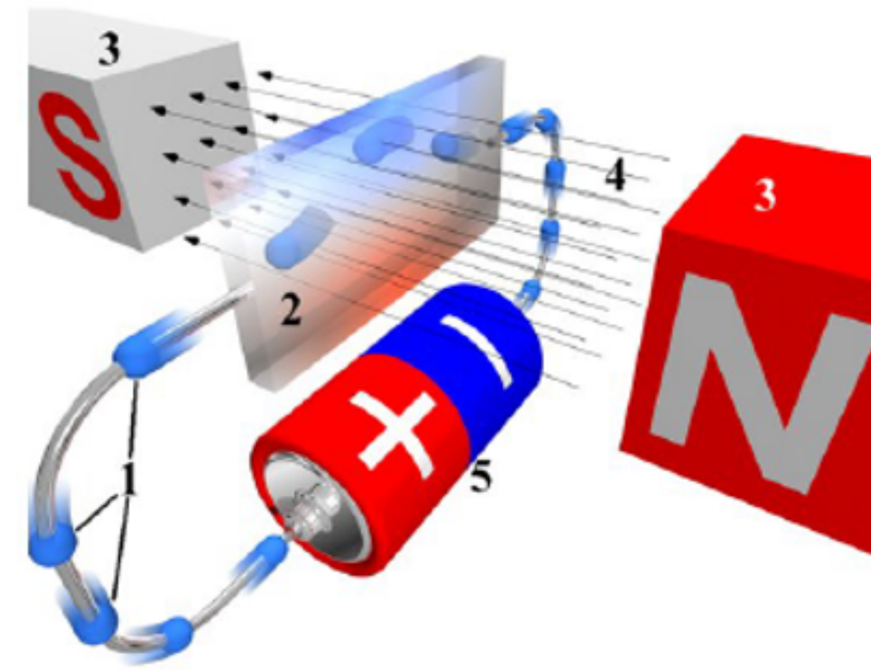
Modified magnetic field evolution

- **Neutron star interior:** complex multi-fluid system.
- **Crust:** solidifies early; nuclei mobility is limited; conductivity governed by electrons.
- **Core:** remains a full multi-fluid on long timescales.
- **Limit:** modified Hall-MHD (eMHD) used in the **crust**





Perfect conductor B.C. at the crust-core interface and potential B.C. at the surface.

$$\frac{\partial \boldsymbol{B}}{\partial t} = -\nabla \times \left[\eta \nabla \times \boldsymbol{B} - \eta k_5 \boldsymbol{B} + f_h (\nabla \times \boldsymbol{B}) \times \boldsymbol{B}\right]$$

$$\left(\eta = \frac{c^2}{4\pi\sigma_e}; \qquad k_5 = \frac{4\alpha\mu_5}{\hbar c}; \qquad f_h = \frac{c}{4\pi e n_e} \right)$$

Ohmic term $\propto (\eta \nabla \times \boldsymbol{B})$**:** the magnetic diffusivity is sensitive to temperature evolution and electron density (strong radial gradients).

Chiral term $\propto (\eta k_5 \boldsymbol{B})$**:** drives exponential growth in some spatial modes by drawing energy from others.

Hall term $\propto (f_h (\nabla \times \boldsymbol{B}) \times \boldsymbol{B})$**:** It naturally creates magnetic discontinuity and transfers energy between different scales.

$$k_5 = \frac{(\nabla \times \boldsymbol{B}) \cdot \boldsymbol{B}}{\dfrac{\mu_e^2}{8\pi\alpha^2\eta(\hbar c)}\Gamma_f + B^2} = \frac{(\nabla \times \boldsymbol{B}) \cdot \boldsymbol{B}}{\left(\dfrac{2\mu_e^2}{m_e^2 c^4}\right)\dfrac{B_{\rm QED}^2}{3\pi} + B^2}$$

$B_{\rm QED} \equiv m_e^2 c^3/(e\,\hbar) = 4.41 \times 10^{13}$ G is the Schwinger QED critical field.

Microphysics: $$B_{\rm sat} \equiv \sqrt{\frac{2}{3\pi}\frac{\mu_e}{m_e c^2}} B_{\rm QED}$$

$B_{\rm sat} \sim 10^{14}$ G (surface) and up to
$B_{\rm sat} \sim 5 \times 10^{15}$ G (inner crustal layers).

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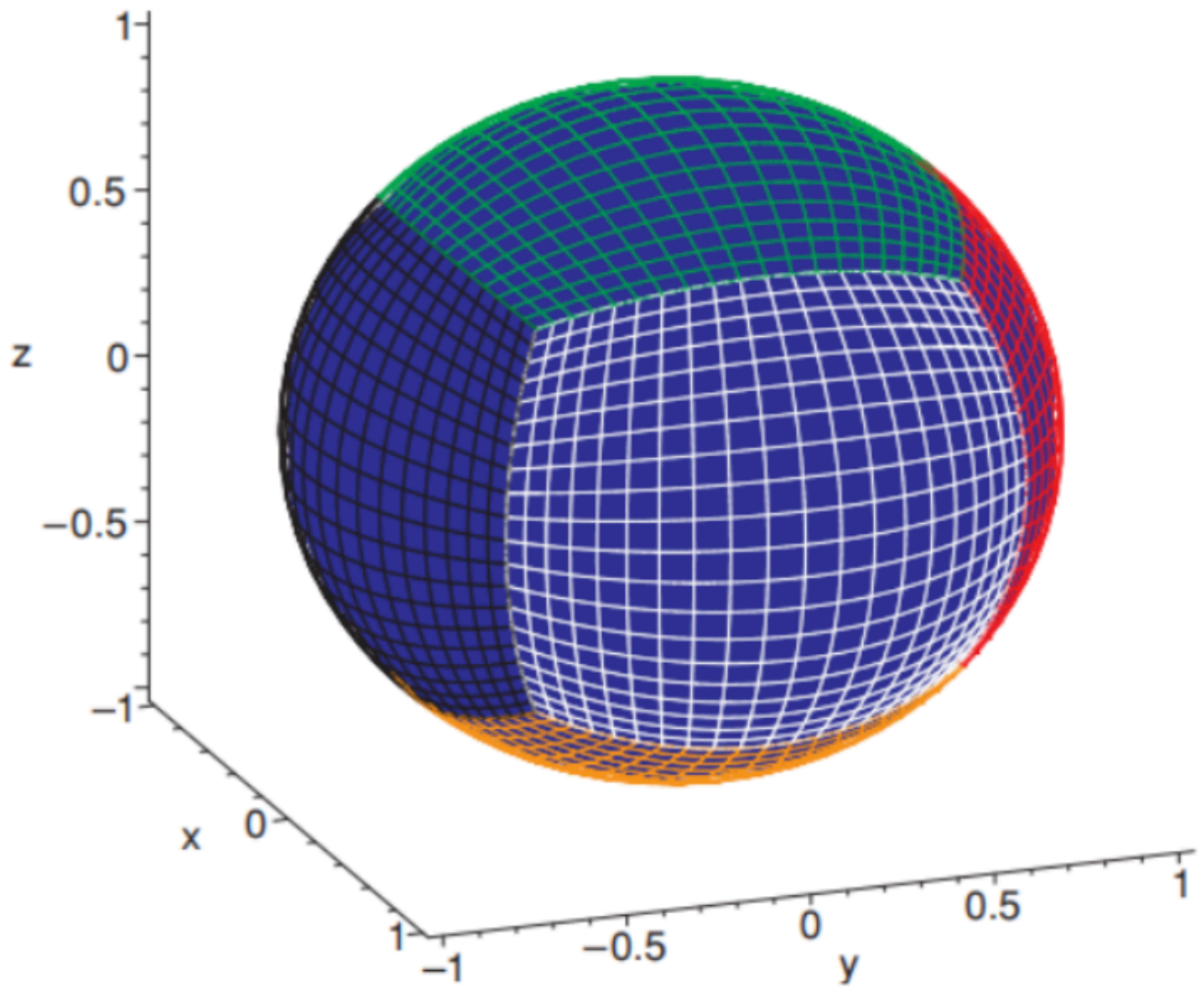
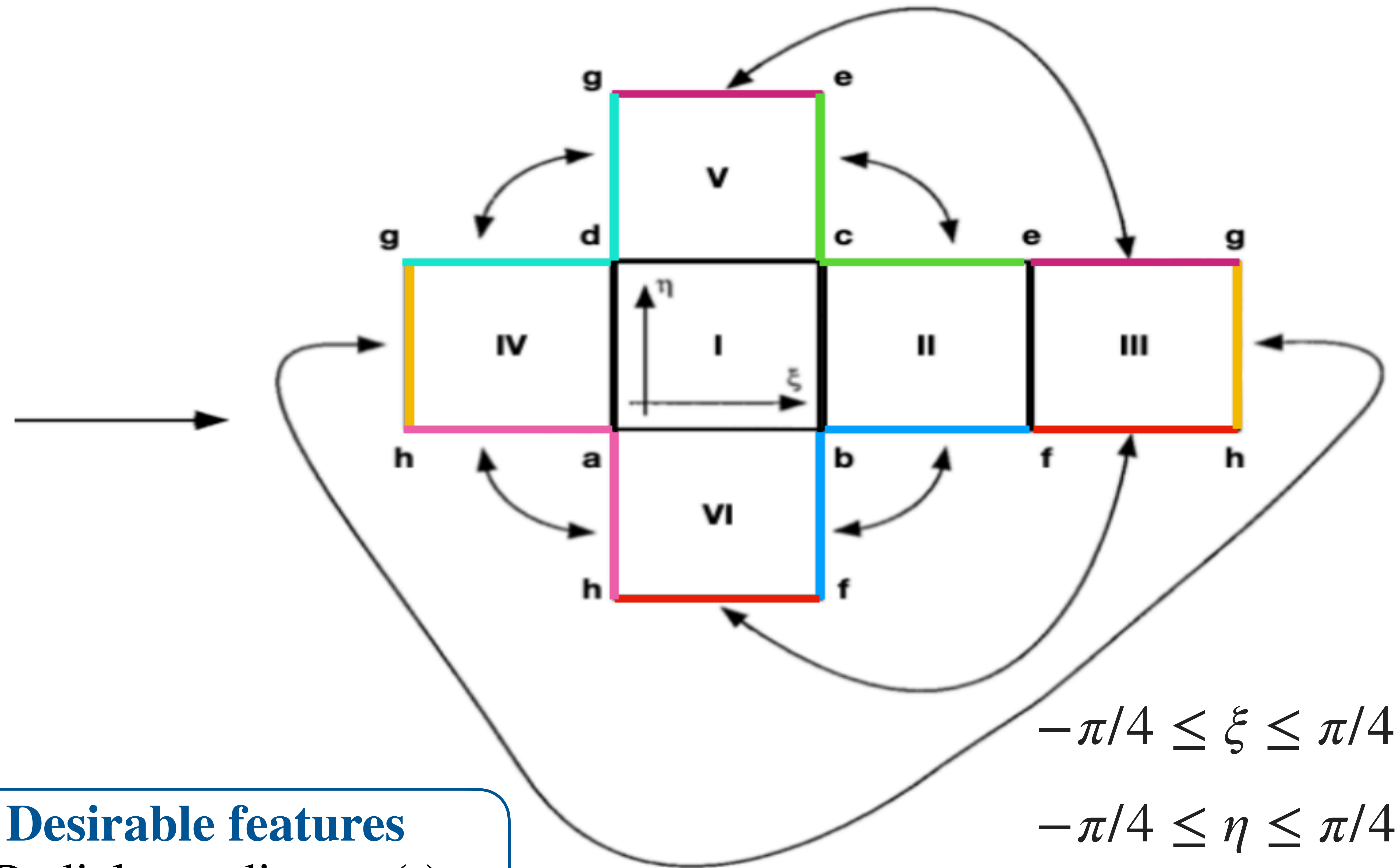
[Dehman & Pons, 2505.06196]

Cubed-sphere coordinates

In 3D spherical coordinates if you want to use **finite-volume**/difference methods, the axis is a singularity. The cubed sphere coordinates are a widely used solution, used in climate and atmospheric simulations



[Ronchi et al. 1996]



$$-\pi/4 \leq \xi \leq \pi/4$$

$$-\pi/4 \leq \eta \leq \pi/4$$

$$g_{ij} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -\frac{X(\xi)Y(\eta)}{C(\xi)D(\eta)} \\ 0 & -\frac{X(\xi)Y(\eta)}{C(\xi)D(\eta)} & 1 \end{pmatrix}$$

non-orthogonal coordinate system

Desirable features

- Radial coordinates (r)
- No axis-singularity
- GR correction

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[Dehman et al. 2023 MNRAS]

(Modified) MATINS the brand new 3D code

$$\frac{\partial \boldsymbol{B}}{\partial t} = -\nabla \times \left[\eta \nabla \times (e^{\nu}\boldsymbol{B}) - \eta k_5 \boldsymbol{B} + f_h \nabla \times (e^{\nu}\boldsymbol{B}) \times \boldsymbol{B}\right] \qquad c_V(T)\frac{\partial (Te^{\nu})}{\partial t} = \vec{\nabla} \cdot (e^{\nu}\hat{\kappa} \cdot \vec{\nabla}(e^{\nu}T)) + e^{2\nu}(Q_J - Q_\nu)$$

Dehman**, Viganò, Pons & Rea 2023, MNRAS* (DOI: **10.1093/mnras/stac2761**): ***Cubed-sphere grid + Magnetic formalism

Dehman**, Viganò, Ascenzi, Pons & Rea 2023, MNRAS* (DOI: 10.1093/mnras/stad1773): **First *3D magneto-thermal simulation

*Ascenzi, Viganò, **Dehman**, Pons & Rea, Perna 2024, MNRAS* (DOI: **10.1093/mnras/stae1749**): ***Thermal formalism***

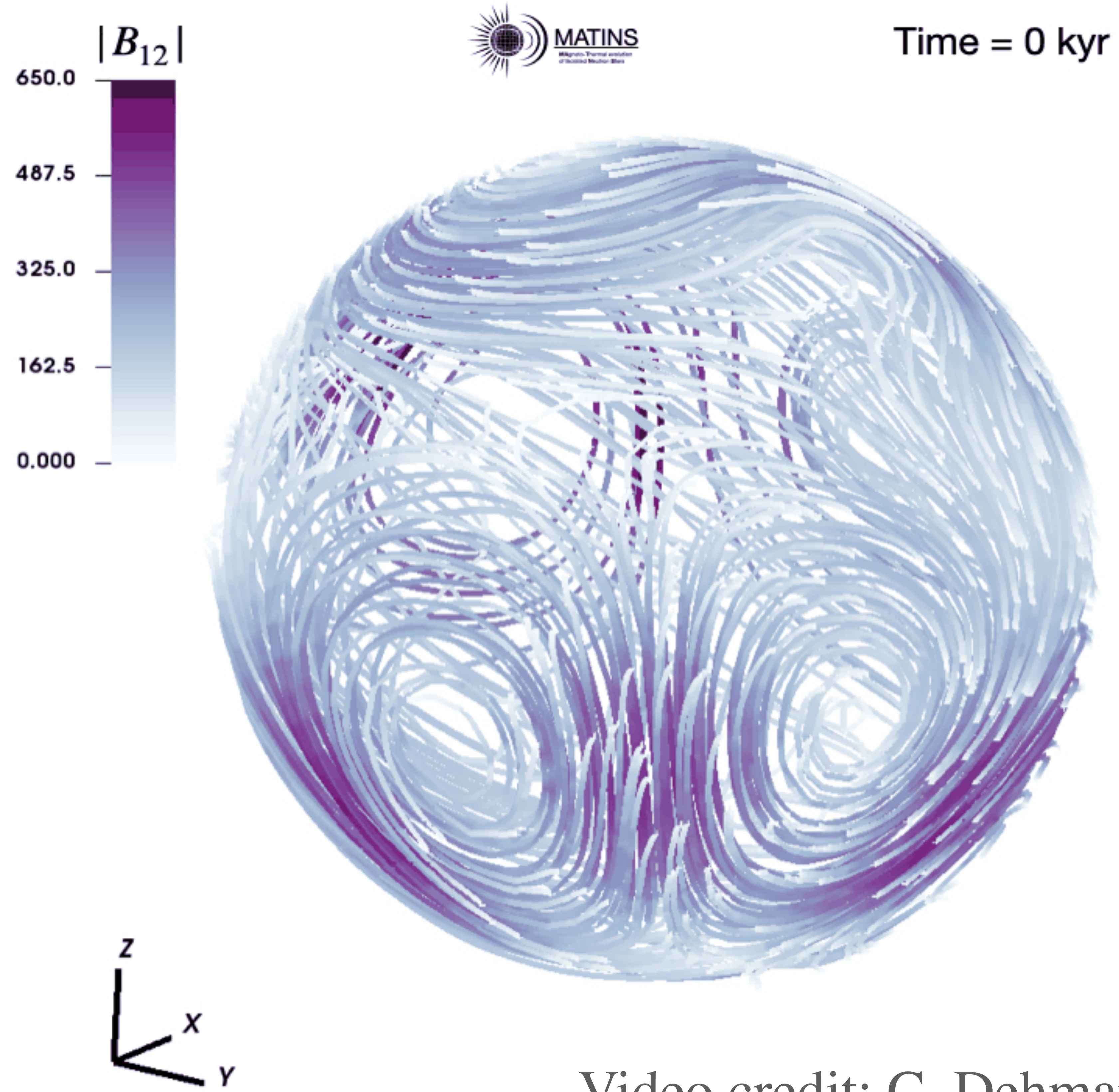
Dehman** & Pons 2025, submitted*: ***Chiral magnetic effect

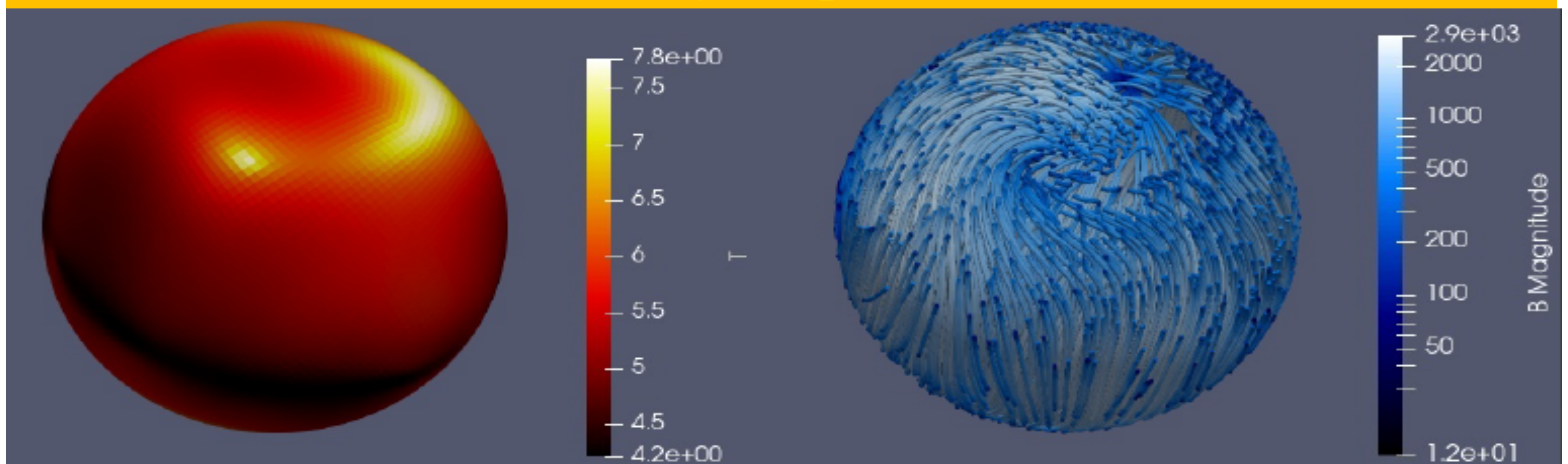
What's better than 2D (Viganò et al. 2021):

- Simulation of 3D magnetic modes, hotspots, and light curves
- Better documentation, **use of novel coordinates (cubed-sphere)**
- Optimization and use of OpenMP

Advance obtained:

- **Realistic 3D evolution and topology, appearance of hotspots**
- **State-of-the-art microphysics and realistic structure**
- **Numerical scheme to better capture non-linear dynamics**
- **General relativistic correction**
- **State of art envelope model**
- **Flexibility in implementing new physics**
- **Documentation and modularity (for public)**





Video credit: C. Dehman

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Initial magnetic field

Requirements:

- Magnetic field with encoded magnetic helicity
- $B \gg B_{\mathrm{QED}} = 4.41 \times 10^{13}\,\mathrm{G}$ — magnetar-like field strength (otherwise too slow)
- Strong $\left(\approx 10^{15} \ldots 10^{16}\,\mathrm{G} \right)$ & turbulent small-scale (tens of meters) magnetic structures.

$$\vec{B} = \vec{\nabla} \times \vec{A} \quad (\vec{k} = \vec{r} = r\hat{e}_r)$$

$$\vec{B}_p = \vec{\nabla} \times \left(\vec{\nabla} \times \Phi \vec{k} \right) \longrightarrow \Phi = \frac{1}{r} \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \Phi_{\ell m}(r) Y_{\ell m}(\theta, \phi) \quad \text{Poloidal}$$

$$\vec{B}_t = \vec{\nabla} \times \Psi \vec{k} \longrightarrow \Psi = \frac{1}{r} \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \Psi_{\ell m}(r) Y_{\ell m}(\theta, \phi) \quad \text{Toroidal}$$

$$\Phi_{\ell m}(r) = \Phi^0_{\ell m} k_r r \left(a + \tan(k_r R) b \right)$$

Linking the toroidal scalar function to the poloidal one (magnetic helicity):

$$\Psi_{\ell m} = \alpha_{\ell m} \Phi_{\ell m}, \quad \text{where } \alpha_{\ell m} = \frac{\sqrt{\ell(\ell+1)}}{R} \longrightarrow \text{Partially helical magnetic field}$$

For a maximally helical field: $\alpha_{\ell m} = k = k_{\mathrm{ang}} + k_r$

$$k\,|H_M(k)|/2E_M(k) \leq 1; \quad H_M = 2E_m(k)/k$$

For CME, radial gradients are key to the dynamics. This is because CME behavior is driven by microphysics in the star (e.g., η, μ_e), which can change sharply within just ~1 km radial layers.

$$k_r \gg k_{\mathrm{ang}}, \qquad k \approx k_r$$

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[Dehman et al. 2023 MNRAS; Dehman & Brandenburg 2025; Dehman & Pons 2505.06196]

Reality of inverse cascade in neutron star crusts

Maximally helical (e.g., positive helicity):

$$k = p + q$$

$$k\,|H_M(k)|/2E_M(k) \leq 1$$

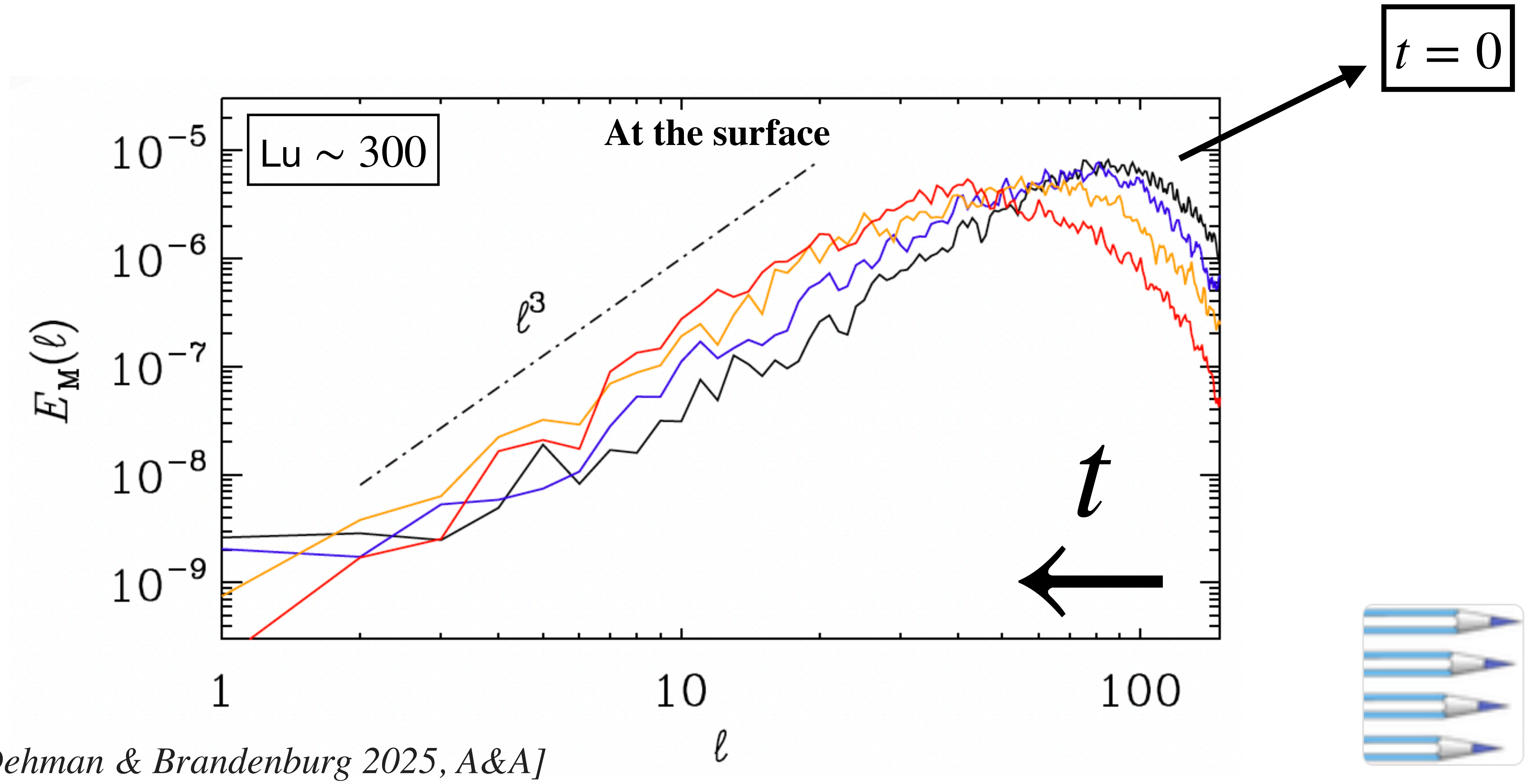
$$|\boldsymbol{k}| \leq \max\left(|\boldsymbol{p}|, |\boldsymbol{q}|\right)$$

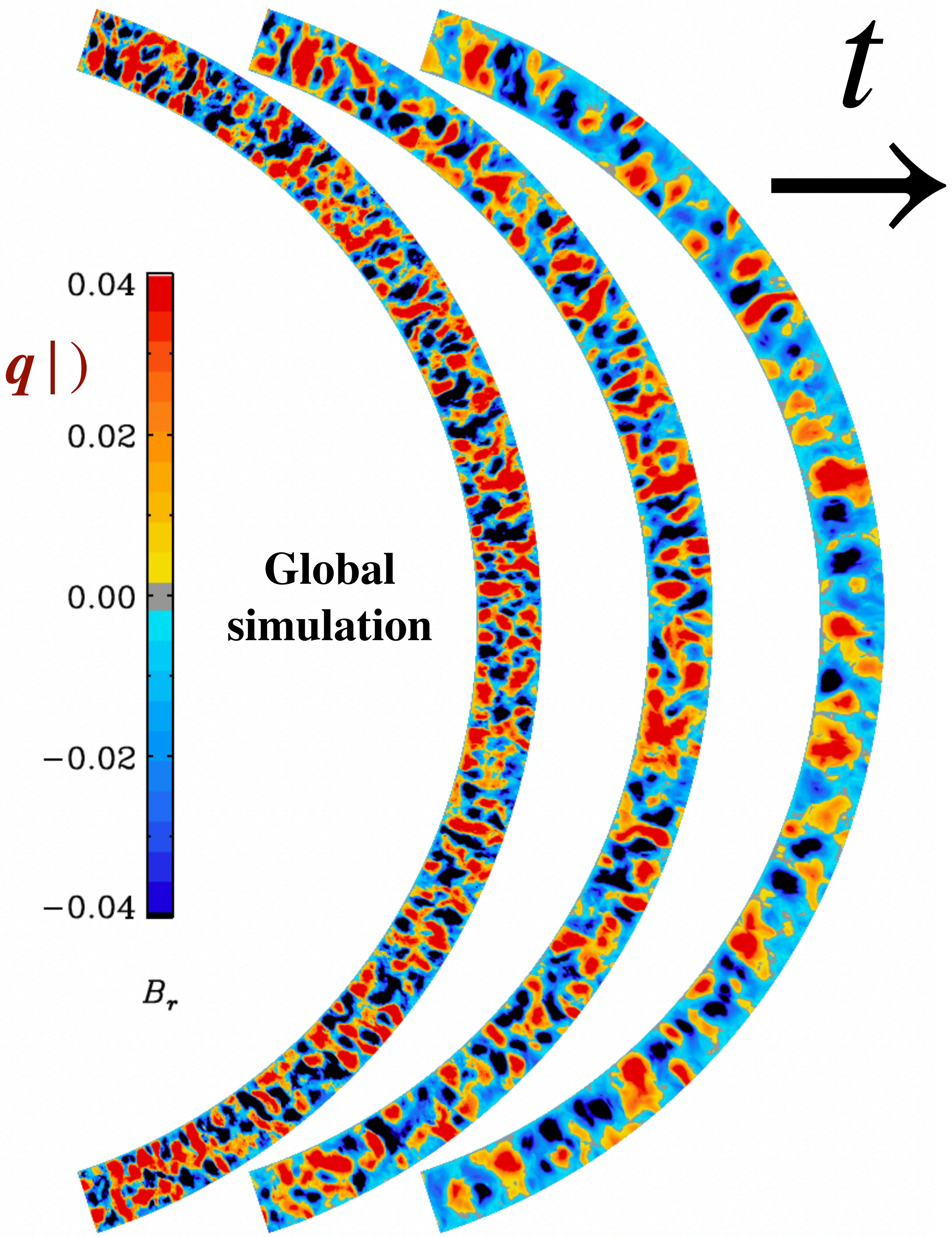
Initial field:

Helical magnetic field.
Random initial field peaking at $l_0 \sim 100$.
Causal spectrum as used in the cosmological context.
Correct aspect ratio of the NS crust.

Inverse Cascade occurs!

Energy transferred from small to large-scale multipoles.
Not observed in previous neutron star simulation studies.
Extreme aspect ratio (1:30)—thin crust— limits the inverse cascade.





[Dehman & Brandenburg 2025, A&A]

Pencil Code

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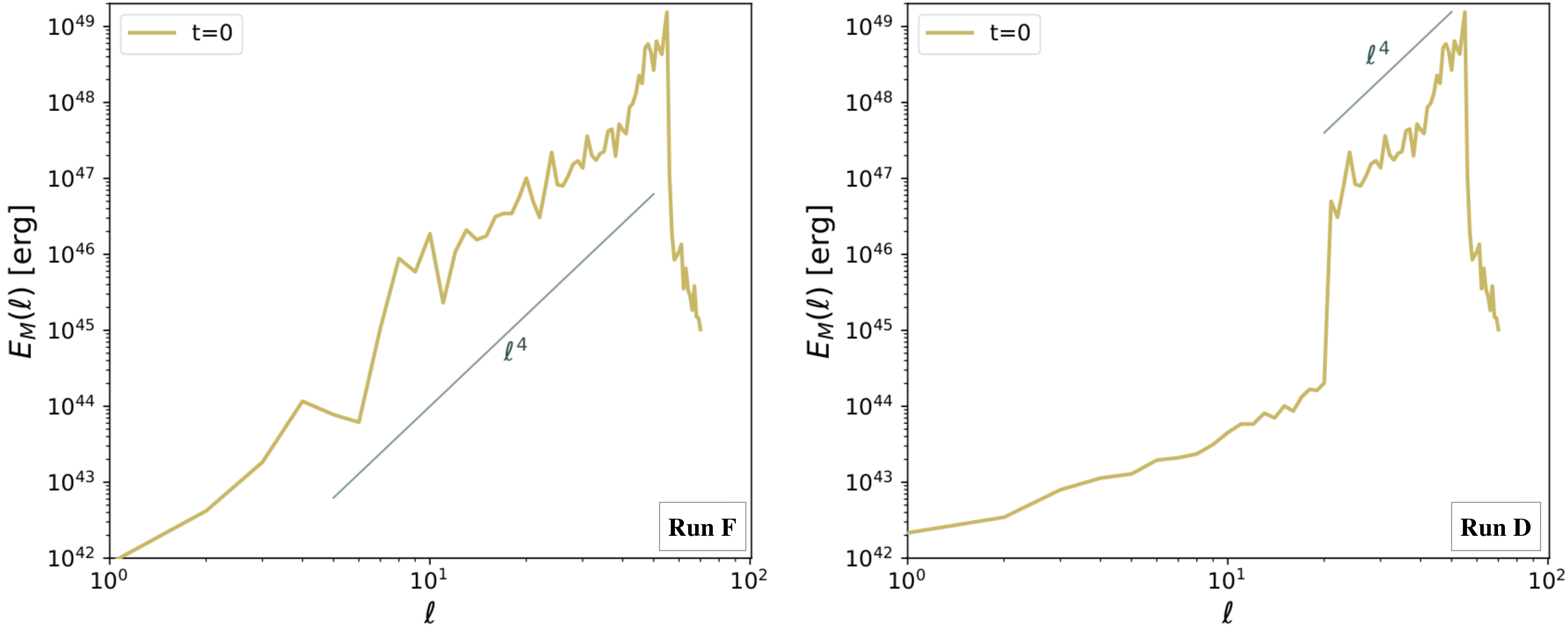
Chiral Magnetic Effect: Magnetic Energy Transfer to Large Scales

— Deliberately excluding Hall term —



- CME redistributes energy toward initially weak large-scale multipoles $(\ell \leq 20)$.
- Strong dipolar $(\ell = 1)$ amplification: natural formation of large-scale structures (more resistant to dissipation).
- Both runs converge to similar spectra, and **modes are saturating at a given field strength.**
- Small-scales dissipate over kiloyear timescales $\tau_{\rm Ohm} = 1/\eta k^2$, leaving large-scale fields intact.
- CME-driven evolution differs from **inverse cascade; no shift of spectral peak to lower ℓ**).

[Dehman & Pons, 2505.06196]

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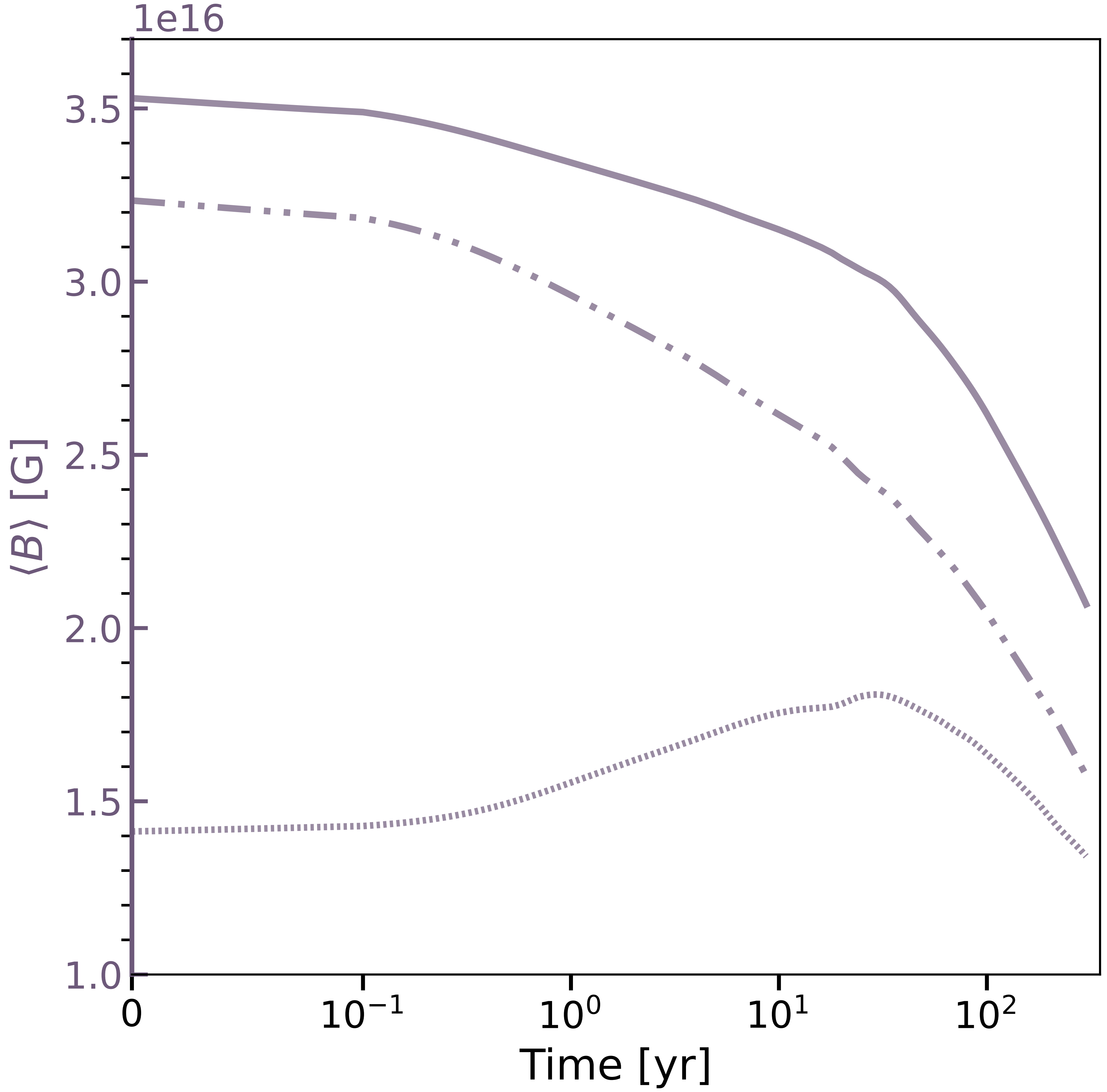
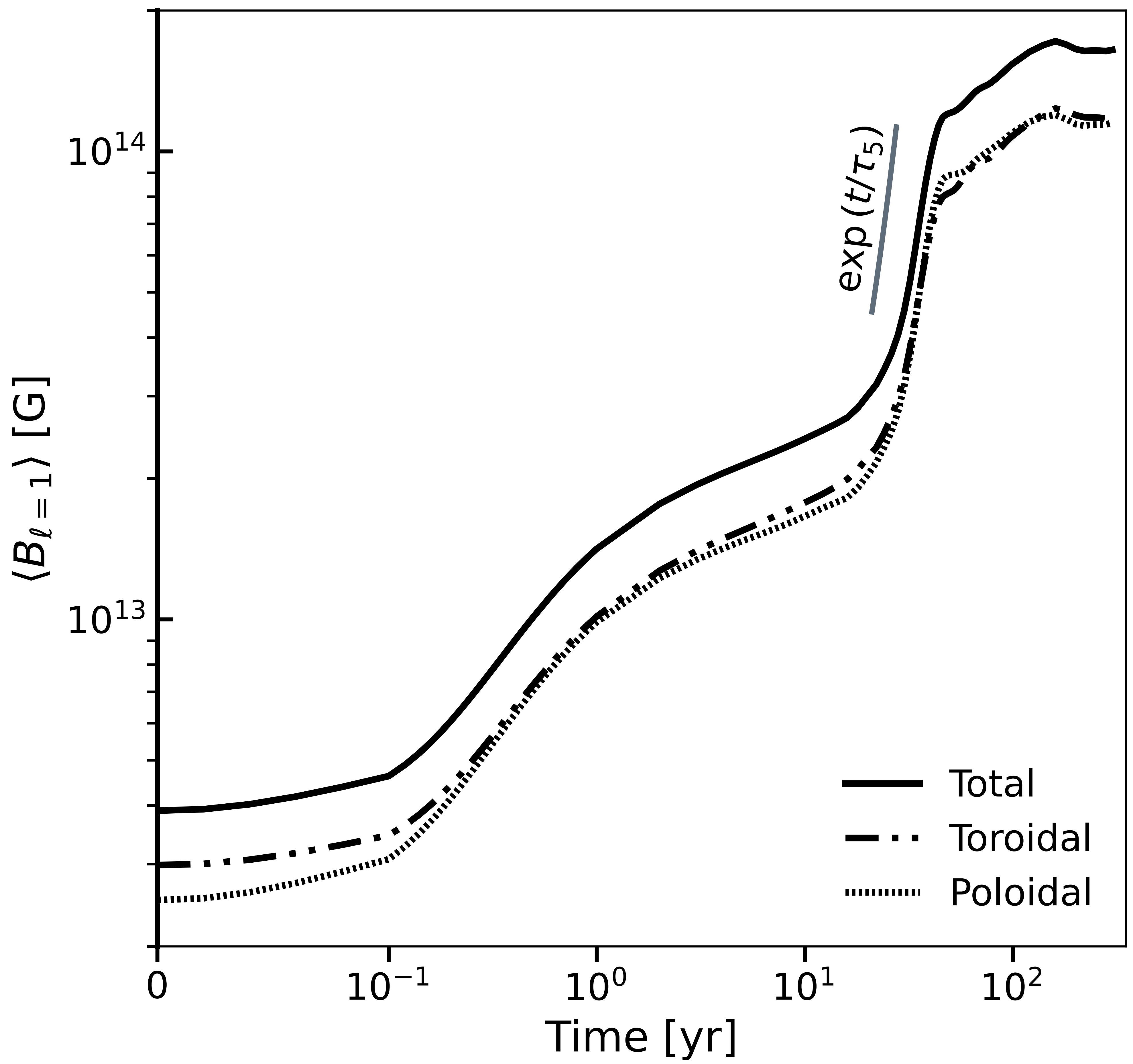
Decay of Average Field & Growth of Dipolar Component

— Three distinct stages —





Poloidal: $$\frac{\partial \Phi_{\ell m}}{\partial t} = \eta\ \Delta\Phi_{\ell m} + \eta\, k_5\, \Psi_{\ell m},$$

Toroidal: $$\frac{\partial \Psi_{\ell m}}{\partial t} = \eta\ \Delta\Psi_{\ell m} - \eta\, k_5\, \Delta\Phi_{\ell m}\,.$$

where, $\Delta \equiv \left(\frac{\partial^2}{\partial r^2} - \frac{\ell\,(\ell+1)}{r^2}\right) \rightarrow -\,k^2$

- CME couples to all (ℓ, m) modes of the poloidal & toroidal fields.
- Mutual Generation: Poloidal ↔ Toroidal fields → drives magnetic energy equipartition.
- Coupling between poloidal & toroidal is asymmetric.

[Dehman & Pons, 2505.06196]

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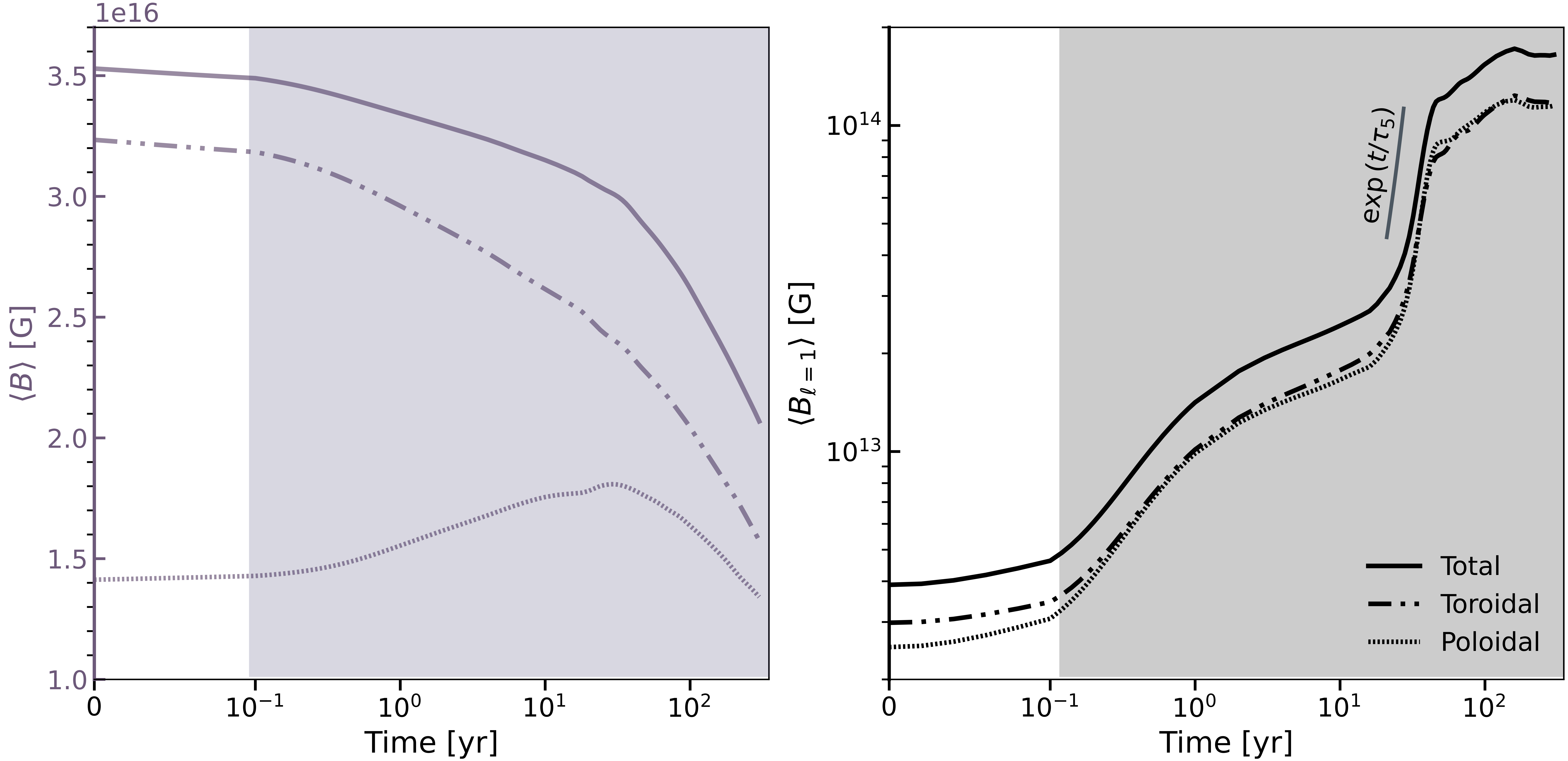
Decay of Average Field & Growth of Dipolar Component

— Early stage ($t \lesssim 0.1$ yr)**—**



Early stage ($t \lesssim 0.1$ yr):

- Total & dipolar fields nearly constant — CME still building up;
- chiral asymmetry not yet dynamically active.

[Dehman & Pons, 2505.06196]

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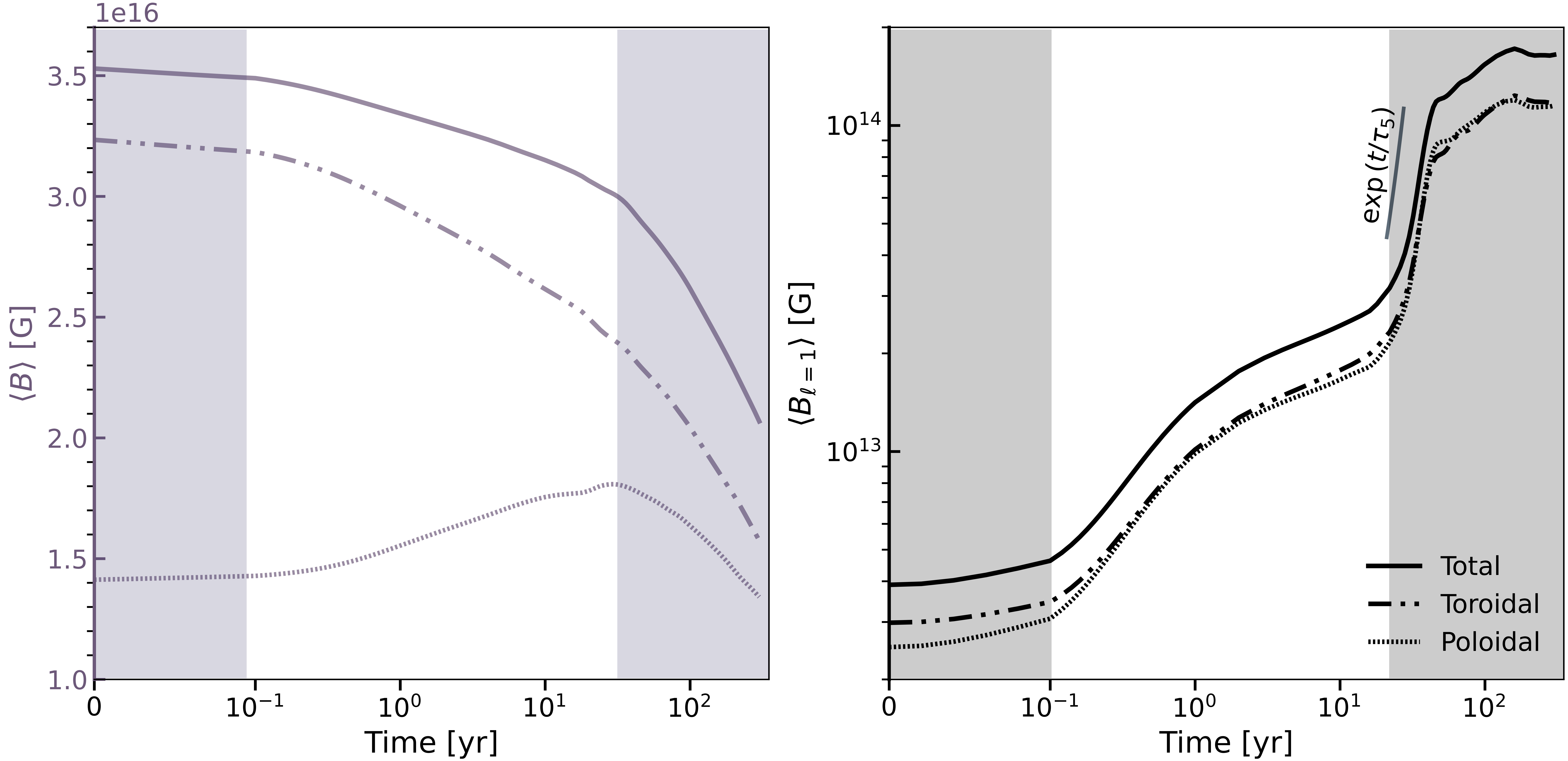
Decay of Average Field & Growth of Dipolar Component

— Intermediate stage ($t \lesssim 30$ yr) —



<u>Intermediate stage ($t \lesssim 30$ yr):</u>

- Total field starts declining (toroidal dissipation).
- CME activates → transfers energy to poloidal field.
- Dipolar components grow with near-equipartition.

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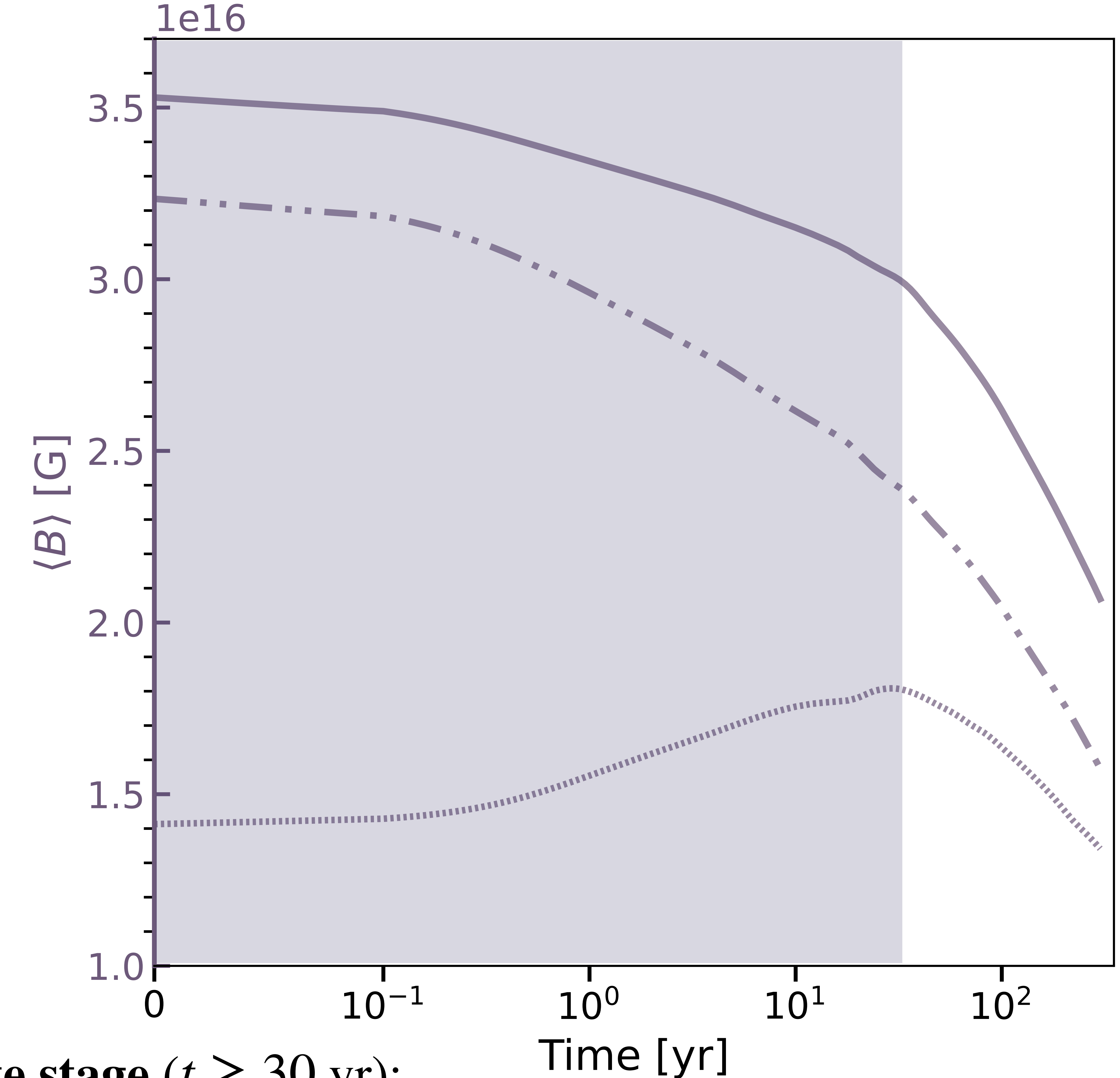
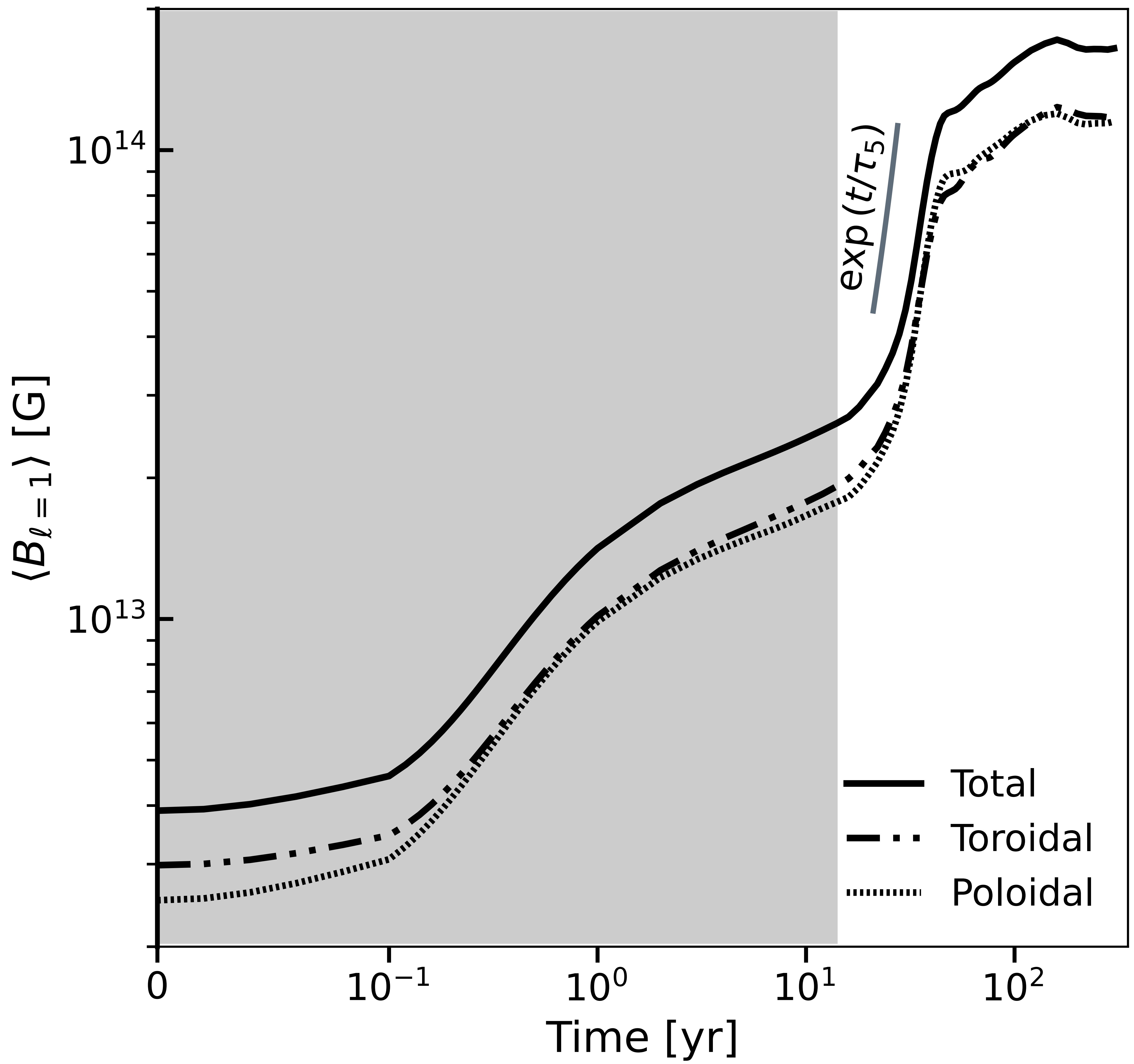
Decay of Average Field & Growth of Dipolar Component

— Late stage ($t \gg 30$ yr) **—**





<u>**Late stage** ($t \gtrsim 30$ yr):</u>

- Poloidal growth halts; both total field components decay similarly.
- Dipolar field grows exponentially ($\propto \exp(t/\tau_5)$, $\tau_5 \approx 5 \ldots 10$ yr) → signals CMI onset.
- Dipole components reach 10^{14} G, then saturates ~100 yr.
- CMI is key to forming the large-scale dipole.

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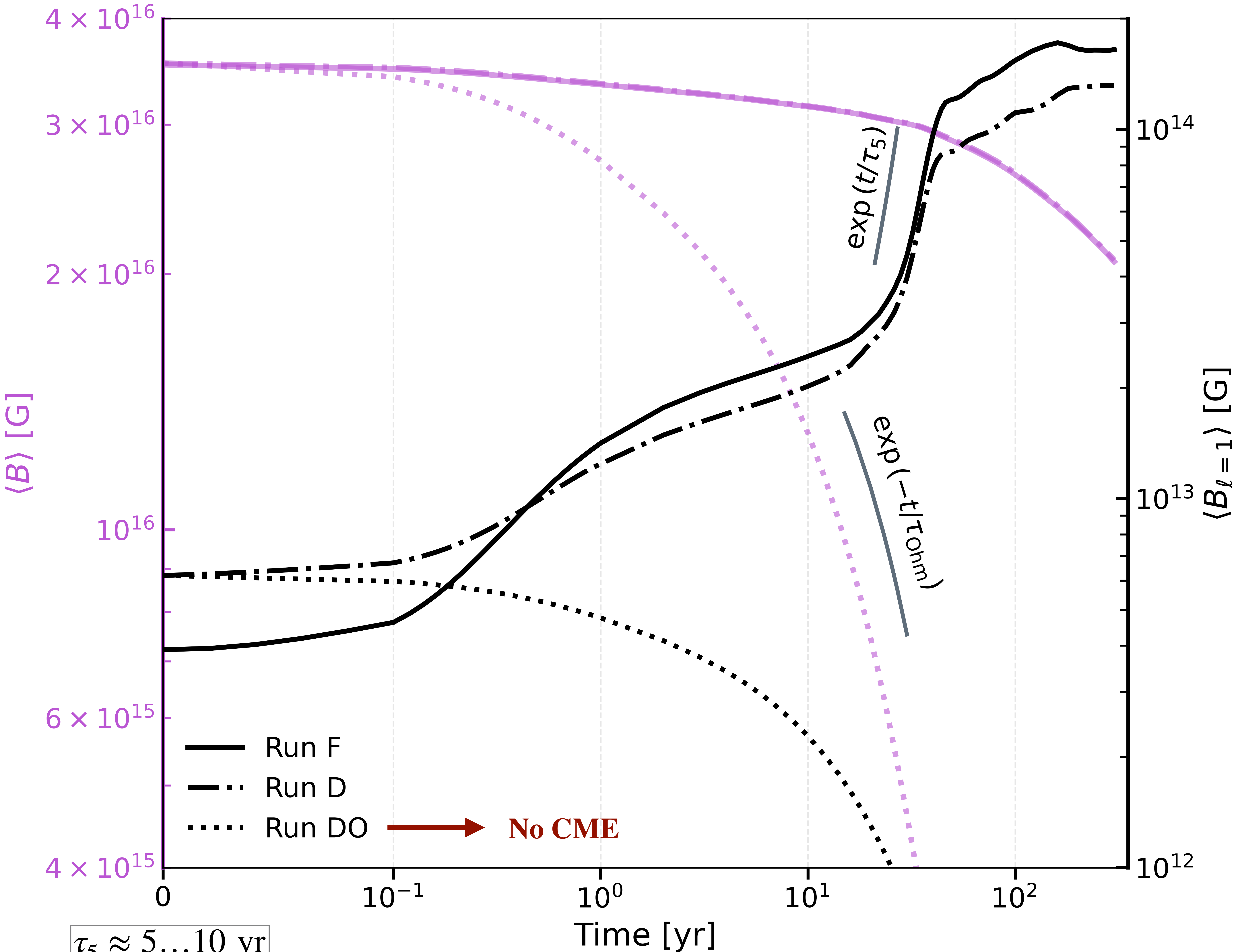
Decay of Average Field & Growth of Dipolar Component

— Pure Ohmic Model vs. CME Dynamics —



$\tau_5 \approx 5 \ldots 10$ yr

$\tau_{\rm Ohm} \approx 20 \ldots 25$ yr

In this case, CME slows down field dissipation:

$$Q_{\rm tot} = \int \sigma_e \boldsymbol{E}^2 dV \,.$$

In the absence of Hall terms:

$$c\,\boldsymbol{E} = \eta\left(\nabla \times \boldsymbol{B} - k_5 \boldsymbol{B}\right)$$

Modified Joule term $\left(-k_5 \boldsymbol{B}\right)$ can suppress, enhance or cancel dissipation.

[Dehman & Pons, 2505.06196]

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Energy Conservation: Electromagnetic & Chiral Imbalance

Electromagnetic energy:

$$\frac{d\varepsilon_{em}}{dt} = -\sigma_e \boldsymbol{E}^2 - \frac{\alpha\mu_5}{\pi\hbar}\boldsymbol{E}\cdot\boldsymbol{B} - \frac{c}{4\pi}\nabla\cdot(\boldsymbol{E}\times\boldsymbol{B}),$$

Electron energy from chiral imbalance:

$$\frac{d\varepsilon_5}{dt} = -\frac{1}{2}\mu_5 n_5 \Gamma_f + \frac{\alpha\mu_5}{\pi\hbar}\boldsymbol{E}\cdot\boldsymbol{B}$$

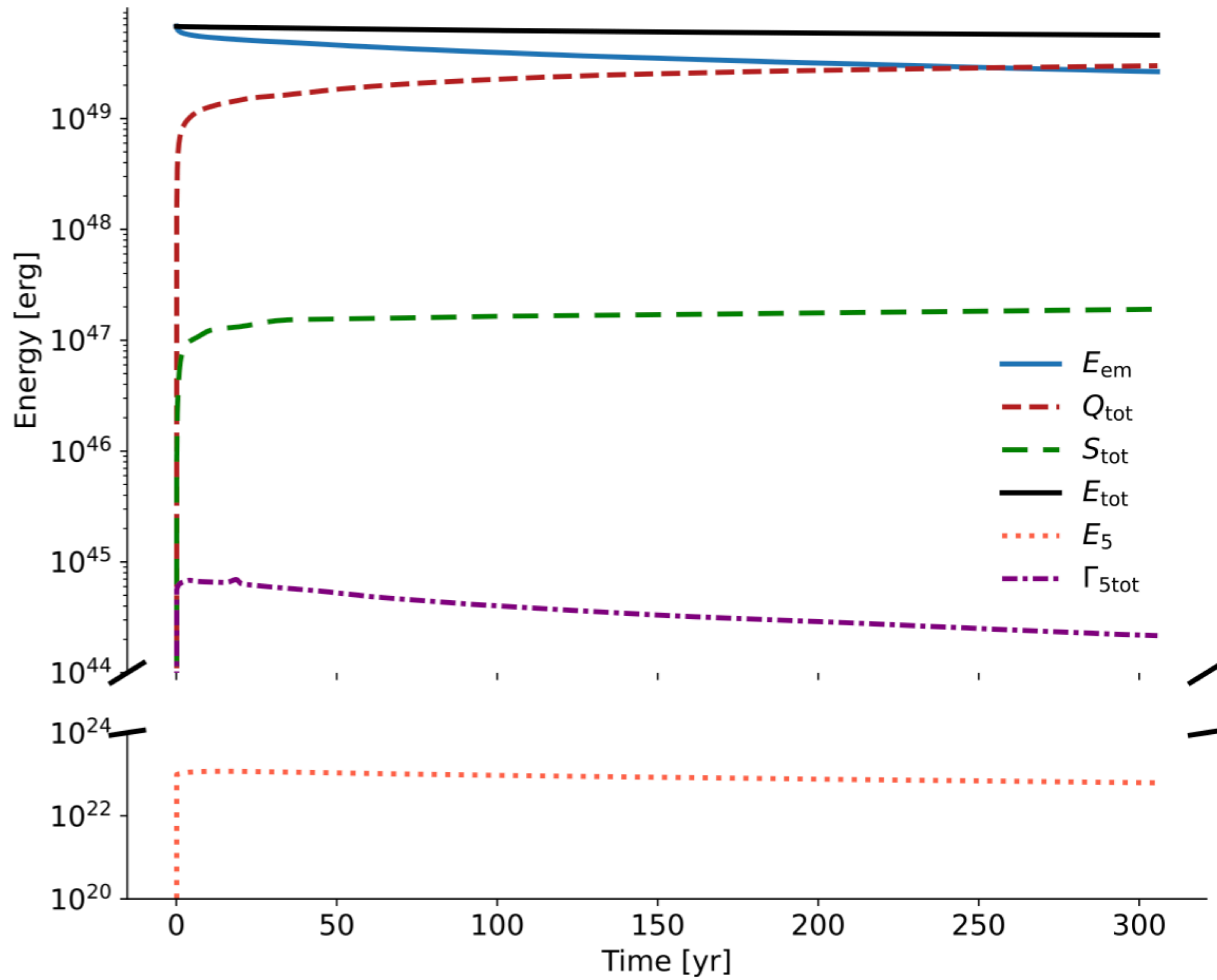
Integrating over the stellar volume, the total energy balance reads:

$$\frac{d}{dt}\left(E_{em}+E_5\right) + S_{\rm tot} + Q_{\rm tot} + \Gamma_{5_{\rm tot}} = 0,$$

Poynting flux: $S_{\rm tot} = \frac{c}{4\pi}\oint dS\cdot(\boldsymbol{E}\times\boldsymbol{B})$

Total spin-flip dissipation rate: $\Gamma_{5_{\rm tot}} = \frac{1}{2}\int \mu_5 n_5 \Gamma_f \, dV$

Joule dissipation: $Q_{\rm tot} = \int \sigma_e \boldsymbol{E}^2 dV$, with $c\boldsymbol{E} = \eta\left(\nabla\times\boldsymbol{B} - k_5\boldsymbol{B}\right)$



- Energy conserved within 5%, with magnetic energy slowly dissipating over time.
- Chiral terms have minor impact, except the one in $Q_{\rm tot}$.

[Dehman & Pons, 2505.06196]

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Summary & Conclusions

Simulations performed with a modified version of MATINS:
a 3D code for magneto-thermal evolution in isolated NS crusts.

Magnetic helicity triggers **chiral asymmetry** in NS crusts (**chiral anomaly**).

CME shapes field evolution over centuries, **overcoming spin-flip suppression**.

Energy transferred from the **small scale structures** $\left(10^{16}\,\mathrm{G}\right)$ toward **larger scales.**
Formation of 10^{14} G dipole (magnetar-like).

Small scales dissipates in a few thousand years explaining magnetar's luminosity $\left(L_X \gtrsim 10^{35}\,\mathrm{erg/s}\right)$

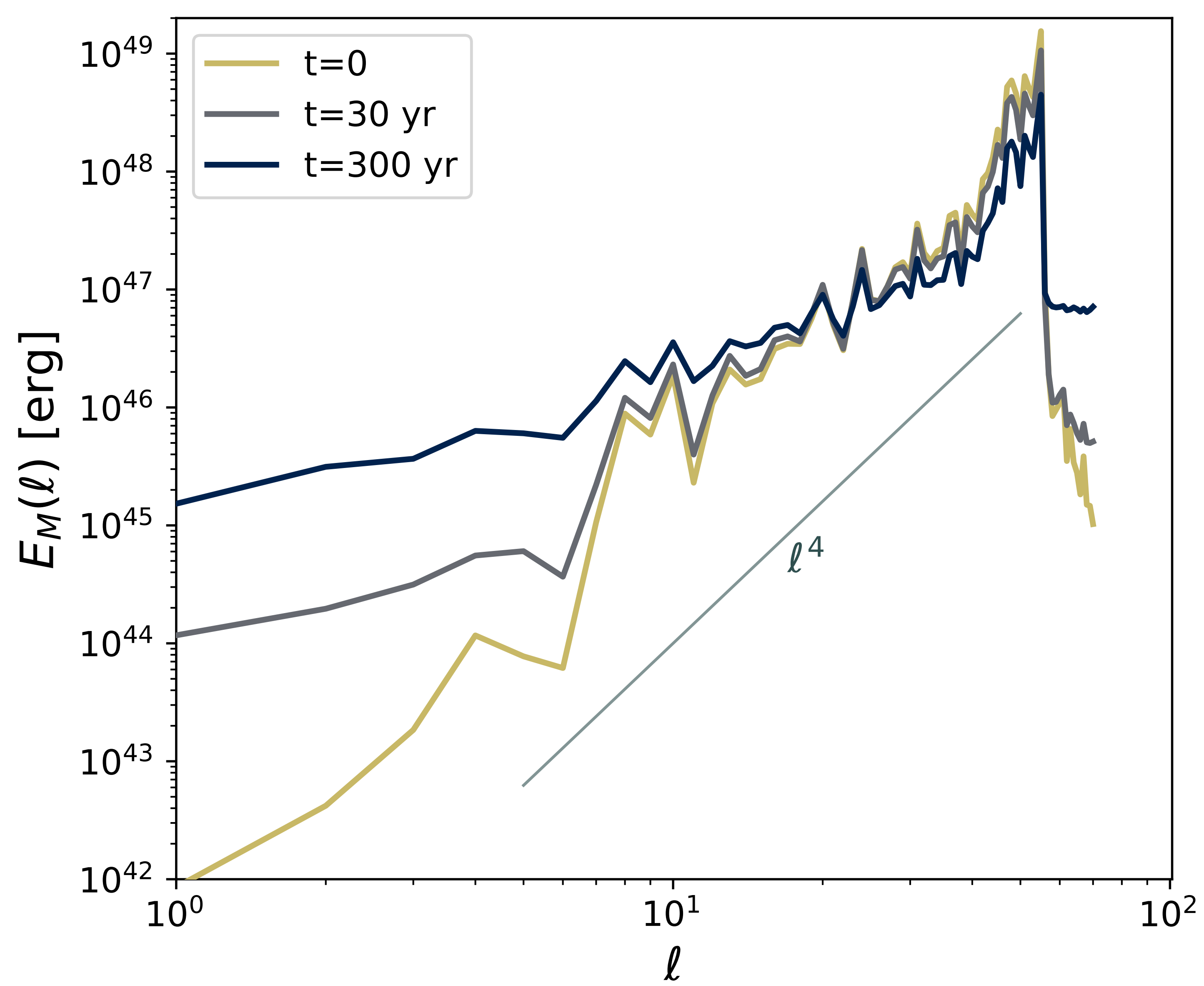
Large scale dipole persists as long-lives structure.

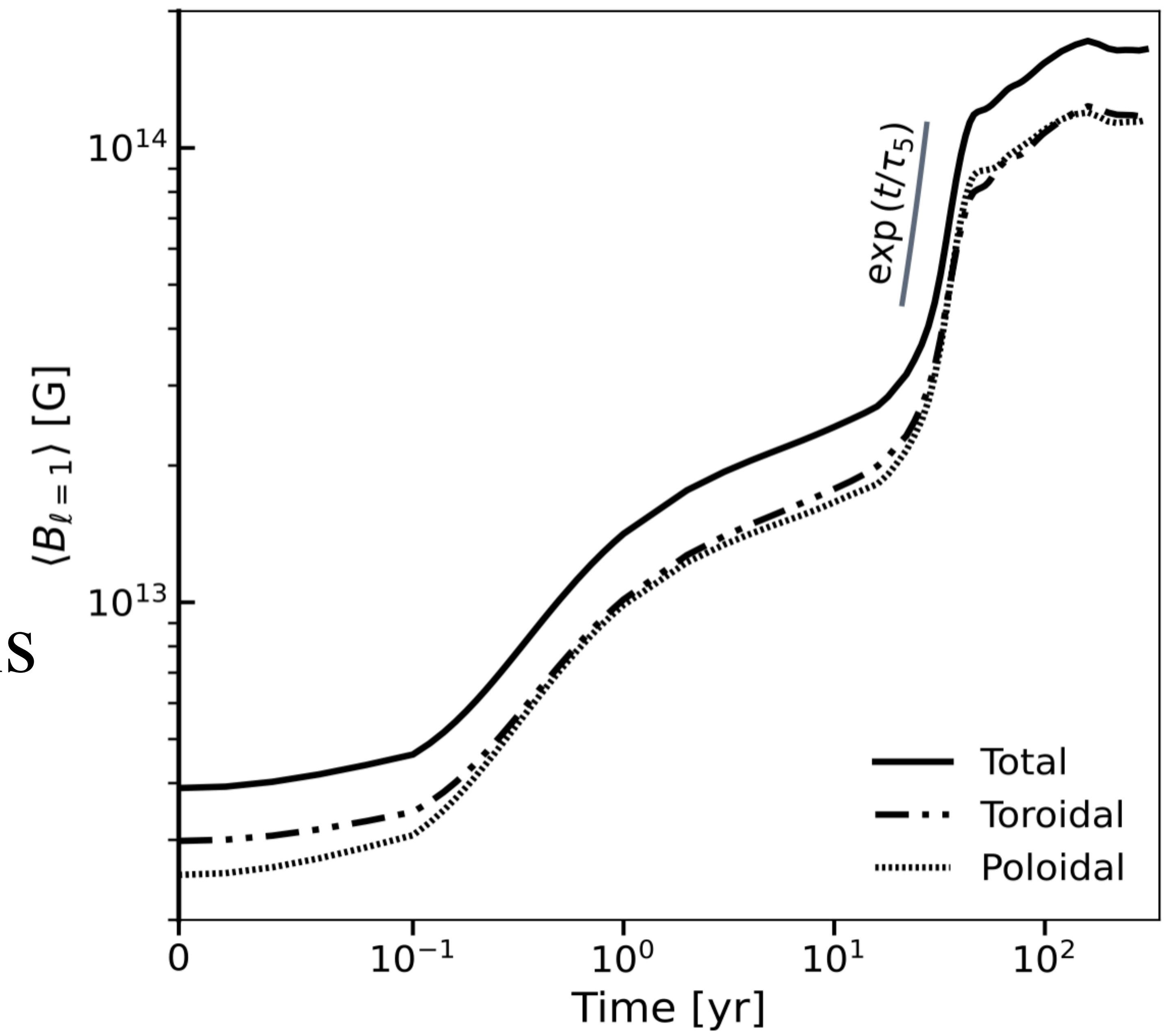
Microphysical mechanism—alternative to traditional hydrodynamic dynamo models—establishing a new framework for **explaining magnetar field dynamics.**

Hall term excluded to isolate CME; limited influence on early-time (100 years).

A lot more can be explored

Questions?





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